Ion Beam Neutral Component Determination by Resonance Radiation Absorption

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The presence of un-ionized propellant efflux from an ion thrustor may be detected by its absorption of resonance radiation characteristic of the atomic species. The absorption of the cesium 8521-Å line and the mercury 2536-Å line are discussed and a transmission curve is predicted for light showing atomic hyperfine structure, line broadening, and source self-absorption. A mercury radiation probe was employed to determine the neutral efflux from a flight-model electron-bombardment engine supplied by NASA Lewis Research Center. A cesium probe was also tested using a simulated cesium ion engine.

Nomenclature

a = quantity defined by Eq. (12)
A = optical absorption
B_{AB} = probability per second per atom of absorbing a quantum of light
B_{EM} = probability per second per atom of induced emission
c = velocity of light
D, P, S = orbital angular momentum designations
E = energy
f = total angular momentum quantum number of an atom
g_i = statistical weight of the ith state
h = Planck's const
h_j = fraction of atoms that can absorb a quantum of wavelength λ_j
I = nuclear spin quantum number
I' = incident light intensity
I_0 = intensity of the mean frequency of a spectral line
$I_{0\nu}$ = incident intensity of light of frequency ν
I_ν = intensity of light of frequency ν
j = total angular momentum quantum number
j = equivalent current density of neutral efflux
J = total angular momentum quantum number
k_ν = absorption coefficient of light of frequency ν
k_0 = absorption coefficient of mean frequency of a spectral line
K_0 = const defined by Eq. (26)
l = electron orbital angular momentum quantum number
L = total orbital angular momentum quantum number
L = length of light path
L_0 = optical path length
M = molecular weight
n = principal quantum number
n = atom density in boiler
N = atomic flux, Eq. (29)
N_T = atoms per unit volume
N_ν = atoms per unit volume capable of absorbing photons of frequency ν
N_0 = atoms per unit area
R = Rydberg const
R_0 = gas const
S = electron spin quantum number
S_g = total area of grid holes
T = absolute temperature
T_i = transmitted light intensity
v = mean molecular velocity
W_0'' = function of electron wave function
X' = quantity defined by Eq. (4)
x = length dimension
Z = distance of light path from the boiler
Ze = nuclear charge
$\bar{\alpha}$ = Sommerfeld fine structure const
α = ratio of emission line width to absorption line width
Δ = measured absorption defined by Eq. (21)
$\Delta\nu_D$ = Doppler line broadening
λ = wavelength
$\bar{\lambda}$ = wave number
λ_0 = mean wavelength of spectral line
ν = frequency
ν_0 = mean frequency of spectral line
τ = lifetime of a quantum state
ϕ = maximum angular width of atomic beam
ω = quantity defined by Eq. (11)

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I. Introduction

RESONANCE absorption techniques have been proposed as a practical means of studying the behavior of ion engines by detecting the neutral atoms in the engine's exhaust plasma. This technique yields an instantaneous and a continuous measure of the neutral atomic density from which the propellant utilization efficiency may be calculated.

This technique requires the incident photon energy to equal the energy difference between two states of the absorbing atoms, usually between the ground state and the lowest excited state. In practice, the resonance condition may be complicated by several physical processes. Atomic spectral lines are often complex, having a detailed structure. The light source may exhibit line broadening and self-absorption. The absorbing atoms may not be in resonance with the incident radiation because of the atoms' velocity, or because of external electric and magnetic fields. Thus, a given photon emitted from an excited atom in a light source, in general, can be absorbed by only a fraction of the neutral atoms of the same element in the exhaust of an ion engine.

These problems are discussed and a transmission versus atom density curve is predicted for radiation exhibiting atomic hyperfine structure, line broadening, and source self-absorption. A lamp and radiation detector system was calibrated by means of an absorption cell for both mercury and cesium. A mercury system was installed on an electron-bombardment ion thruster supplied by NASA Lewis Research Center, and the neutral atom density of the exhaust plasma was measured. A neutral cesium atomic beam from a cesium oven was monitored by means of optical absorption.

II. Atomic Line Spectra

The resonance absorption experimentally observed is, in general, the sum of the absorption of several hyperfine com-

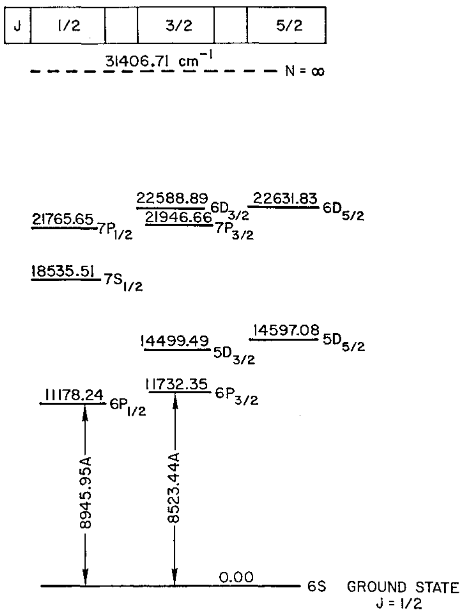

Fig. 1 The energy level structure of cesium.

ponents. Hyperfine structure is determined by the properties of the atomic states and the nucleus. Cesium has a hydrogen-like structure and its spectrum may be understood in terms of the hydrogen system. Mercury, on the other hand, has two optically active electrons and is discussed in terms of helium.

The energy level scheme for hydrogen is given by[1]

$$E = -\frac{Rch}{n^2}\left[1 + \frac{\bar{\alpha}^2}{n}\left(\frac{1}{j+\frac{1}{2}} - \frac{3}{4n}\right)\right] \quad (1)$$

Here, n and j are quantum numbers; n an integer, is called the principal quantum number. For a given $n = n'$, j is the total angular momentum of the electron, the sum of the orbital angular momentum, and the intrinsic electron spin (j may have all half-integral values $j = \frac{1}{2}, \frac{3}{2}, \ldots n' - \frac{1}{2}$). The Sommerfeld fine structure constant $\bar{\alpha}$ has the value $\frac{1}{137}$. The numbers R, c, and h are, respectively, the Rydberg constant, the velocity of light, and Planck's constant. The wave numbers corresponding to the energy levels given by (1) are found by the relation

$$\lambda\!\!\!\bar{} = E/hc \quad (2)$$

The line spectra of the alkali elements are similar to the spectrum of the hydrogen atom. An alkali atom may be pictured as a nucleus surrounded by a symmetrical electron shell plus one outer optically active electron, which is partially shielded from the coulomb charge on the nucleus by the inner electrons. This structure yields an equation similar to that of the hydrogen atom with the nuclear charge Ze, replaced by some effective charge $(Ze)_{\text{eff}}$. However, the degree of shielding by the inner electrons will depend on the outer electron's orbit that, in turn, is represented by the orbital angular momentum of the electron. The energy levels then depend, not only on the quantum number n and j, but also on the orbital angular momentum quantum number l. Figure 1 shows the lowest energy levels for cesium.[2] The orbital angular momentum is indicated by the standard notation $S(l = 0)$, $P(l = 1)$, $D(l = 2)$, ... All levels of a given principal quantum number n, and the same orbital angular momentum l (except $l = 0$), are split into doublets of $j = (l - \frac{1}{2})$ and $j = (l + \frac{1}{2})$. The principal spectral lines (transitions from excited states to the ground state) are then closely spaced doublets.

The energy structure for atoms with two valence electrons is more complicated since there is an additional electron outside the closed electron shell. The simplest example is helium. There exist for helium two series of energy levels; the singlets with $S = 0$ and the triplets with $S = 1$ where S is the sum of the individual electron's spin, $S = s_1 + s_2$. States are labeled according to the principal quantum number of the excited electron; the orbital angular momentum of the electrons ($S = 0$, $P = 1$, $D = 2$, etc.); the total spin (singlet = 1 or triplet = 3); and the total angular momentum $J = 0$, 1, 2, etc.

The elements of Group II of the periodic table, of which mercury is a member, have relatively inert electron shells plus two external optically active electrons. For small atomic numbers, the two external electrons tend to interact in such a way that the intrinsic spins of the two electrons form a total spin S and the orbital angular momenta of the two electrons add to create a total orbital angular momentum L. The energy levels are determined by S and L with the total angular momentum J producing a very small effect on the energy of a state. Transitions between singlet and triplet states are prohibited. As the atomic number increases, the energy dependence due to the total angular momentum J becomes greater. If it is then assumed that the spin and the orbital angular momentum of each active electron combine to form a total angular momentum for that electron in the heavy atoms, then the total angular momentum of the atom is the sum of the individual electrons' total angular momenta. The energy level structure of these atoms may be accounted for by this scheme. Transitions between levels are permitted if the total angular momentum of the electron changes by ± 1 or 0. For mercury, transitions between triplet and singlet states are permitted (see Fig. 2).

III. Hyperfine Structure

Quite often when atomic spectral lines are examined with spectrometers of very high resolution, a previously single fine structure line is found to consist of several, sharp, closely spaced lines. This is called the hyperfine structure. Filters may be obtained which will absorb all but one line of a fine structure group. However, no practical techniques are presently available to permit the transmission of only one hyperfine structure line. This means that, in general, one must deal with several lines rather than one line. Since a photon will be absorbed by only those atoms capable of making the same energy transition as the atom that produced the photon (ignoring the recoil energy of the atom), each line of a hyperfine group can be absorbed by only a part of the atoms of the

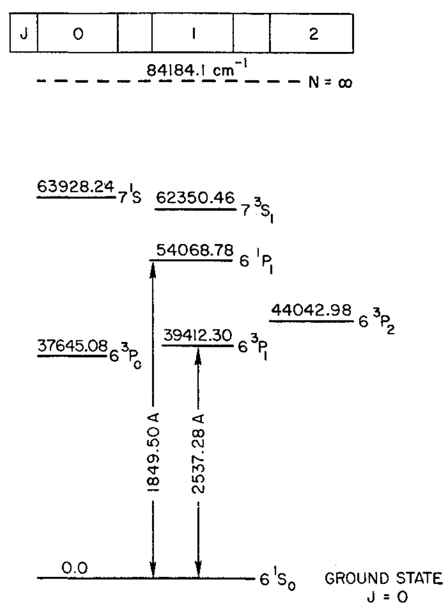

Fig. 2 The energy level structure of mercury.

same element. Thus, the transmission of a spectral line is the sum of the separate hyperfine component transmissions.

Hyperfine structure is caused by the nucleus. If the atoms consist of more than one isotope, or if the nucleus has a magnetic moment, hyperfine structure will occur.

A. Cesium

Cesium has only one stable isotope Cs^{133}. However, it does have a rather large nuclear spin and nuclear magnetic moment. The interaction of the valence electron with the nuclear magnetic moment increases the energy level by an amount[1]

$$\Delta E = W_0'' \left[\frac{f(f+1) - I(I+1) - j(j+1)}{j(j+1)(2l+1)}\right] \quad (3)$$

where

I = nuclear spin = $\frac{7}{2}$ for cesium
j = total angular momentum of the electron
l = orbital angular momentum of the electron
g = total angular momentum of the atom and may have all values from $|I + j|, |I + j - 1| \ldots |I - j|$
W_0'' = a function of the electron wave function and is quite sensitive to the orbital angular momentum of the electron

Let

$$X' = \frac{f(f+1) - I(I+1) - j(j+1)}{j(j+1)(2l+1)} \quad (4)$$

The ground state of cesium has $j = \frac{1}{2}$, $l = 0$ and is split into two levels with

$$f = 4 \qquad X' = \tfrac{14}{3}$$
$$f = 3 \qquad X' = -6$$

The $P_{1/2}$ level has $j = \frac{1}{2}$, $l = 1$ and is split into two energies with

$$f = 4 \qquad X' = \tfrac{14}{9}$$
$$f = 3 \qquad X' = -2$$

The $P_{3/2}$ level has $j = \frac{3}{2}$, $l = 1$, and is split into 4 components:

$$f = 5 \qquad X' = \tfrac{14}{15}$$
$$f = 4 \qquad X' = \tfrac{2}{45}$$
$$f = 3 \qquad X' = -\tfrac{2}{3}$$
$$f = 2 \qquad X' = -\tfrac{6}{5}$$

An additional selection rule permits transition only between energy levels if $\Delta f = 0$ or ± 1.

The 8523-Å line ($P_{1/2} - S_{1/2}$ transition) is split into four distinct spectral lines. The 8946-Å line ($P_{3/2} - S_{1/2}$ transition) becomes 6 lines. The smallest separation of the hyperfine structure lines is estimated to be the same order of magnitude as the expected Doppler broadening.[9] Thus, the 8523-Å line will be treated as 4 distinct lines and the 8946-Å line considered to be 6 individual lines (see Fig. 3).

The relative intensities of the hyperfine components of the 8523-Å and 8946-Å lines of cesium are given in Table 1.

Table 1 Cesium hyperfine structure

Line	Relative Intensity
Cs ($P_{1/2} - S_{1/2}$) Transition	
α	71.5
β	100
γ	100
δ	33.4
Cs ($P_{3/2} - S_{1/2}$) Transition	
α	100
β	48
γ	34
δ	16
ϵ	48
ζ	45

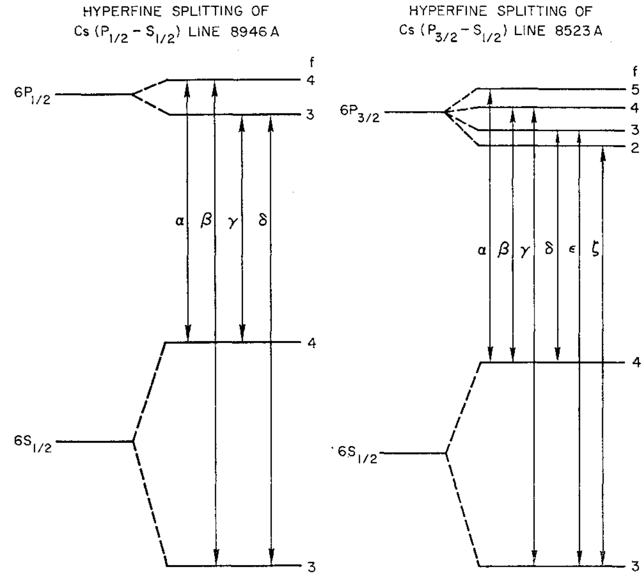

Fig. 3 The hyperfine splitting of the cesium 8946-Å line and the cesium 8523-Å line.

B. Mercury

Natural mercury consists of six principal isotopes.[3] In addition, two of these isotopes Hg^{199} and Hg^{201} have a nuclear spin as shown in Table 2.

The even isotopes produce 4 separate spectral lines about equally spaced of $\Delta\lambda = 180\ cm^{-1}$. The isotope 199 has a nuclear spin of $\frac{1}{2}$, which splits the ($6^3P_1 - 6S$) transition into 2 components, and the isotope 201 has hyperfine structure splitting of 3 lines. One of the Hg^{199} lines and one of the Hg^{201} lines superimpose with the Hg^{204} line. Another of the Hg^{201} lines lies quite close to the Hg^{198} line. The remaining Hg^{201} and Hg^{199} lines superimpose to form essentially another single line. The net result is shown in Fig. 4. The result is a group of five lines of approximately the same intensity.

Table 2 Mercury

Isotope	Abundance	Nuclear Spin
198	0.100	0
199	0.169	$\frac{1}{2}$
200	0.231	0
201	0.132	$\frac{3}{2}$
202	0.297	0
204	0.068	0

IV. Optical Absorption

If parallel light of intensity $I_{0\nu}$ and frequency ν is sent through an absorbing medium of thickness x, the absorption coefficient k_ν, for this light, may be defined by the equation

$$I_\nu = I_{0\nu} e^{-k_\nu x} \quad (5)$$

with I_ν equal to the intensity of the transmitted light.

Consider a light beam (parallel rays) passing through a layer of atoms dx in thickness in which ΔN_ν atoms may absorb a quantum of light of frequency ν, and in which there are

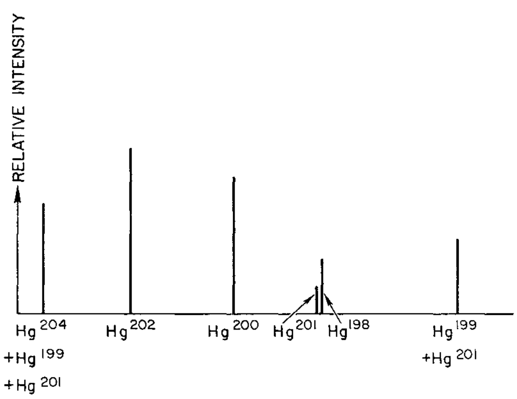

Fig. 4 The hyperfine structure of the mercury 2537-Å line.

$\Delta N_\nu'$ excited atoms that may emit a quantum of this frequency. If spontaneous emission is neglected and if it is assumed that there are a negligible number of excited atoms present, the loss of energy is

$$-d[I_\nu \Delta\nu] = \Delta N_\nu h\nu dx B_{AB} I_\nu (1/4\pi) - \Delta N_\nu' h\nu dx B_{EM} I_\nu (1/4\pi) \quad (6)$$

with B_{AB} the probability per second per atom of absorbing a quantum $h\nu$ and B_{EM} the probability per second per atom of induced emission.

Since $B_{EM}/B_{AB} = g_1/g_2$, the ratio of the statistical weight of the ground state to the excited state of the atom, Eq. (6) becomes

$$-\frac{1}{I_\nu}\frac{dI_\nu}{dx}\Delta\nu = \frac{h\nu}{4\pi}(B_{AB}\Delta N_\nu - B_{EM}\Delta N_\nu') \quad (7)$$

Using the fact that

$$B_{EM} = 2h\nu^3 g_1/g_2\tau c^2 \quad (8)$$

with τ equal to the lifetime of the excited state and assuming that the number of excited atoms is very much smaller than the number in the ground state, then one has

$$\int k_\nu d\nu = \frac{\lambda_0^2 g_2 N_T}{8\pi g_1 \tau} \quad (9)$$

where the integral is carried over the entire frequency distribution of the incident light.

Spectral lines may be distorted or broadened by several processes. These include the natural broadening, Doppler effect broadening, and broadening due to interactions with atoms and electrons. If the gas pressure is below about $\frac{1}{10}$ mm of Hg and there are few free electrons present, the last effect may be ignored.

When both Doppler and natural broadening are important the absorption coefficient is

$$k_\nu = k_0 \frac{a}{\pi}\int_{-\infty}^{\infty}\frac{e^{-y^2}}{a^2 + (\omega - y)^2}dy \quad (10)$$

where

$$\omega = [2(\nu - \nu_0)/\Delta\nu_D](\ln 2)^{1/2} \quad (11)$$

$$a = (\ln 2)^{1/2}/2\pi\tau\Delta\nu_0 \quad (12)$$

with k_0 the absorption coefficient at the center of the line (at frequency ν_0) and

$$\Delta\nu_D = [(8R_0T/M)\ln 2]^{1/2}\nu_0/c \quad (13)$$

Here R_0 is the gas constant, M is the molecular weight and

$$k_0 = \frac{1}{\Delta\nu_D}\left(\frac{\ln 2}{\pi}\right)^{1/2}\cdot\left(\frac{\lambda_0^2 g_2 N_T}{4\pi g_1 \tau}\right) \quad (14)$$

Since $\frac{1}{2}\pi\tau$ is the natural width of the radiation line, the constant a represents the ratio of natural linewidth to the Doppler line broadening and is called the damping ratio. In general, a is very small, of the order of 10^{-2} or less. For small a, $k_\nu = k_0 e^{-\omega^2}$.

When resonance radiation is passed through absorbing material, the ratio of transmitted-to-incident radiation may be measured and the absorption defined

$$A = 1 - T_i/I' \quad (15)$$

where T_i = transmitted radiation intensity and I' = incident radiation intensity.

In general, the frequency distribution of the source of the radiation, the lamp, is not well known and the light shows line-broadening and self-absorption. With a frequency distribution of incident radiation represented by I_ν, the absorption of a gas layer of thickness x and absorption coefficient k_ν becomes

$$A = 1 - \int I_\nu e^{-k_\nu x}d\nu / \int I_\nu d\nu \quad (16)$$

Even if the frequency distribution of the lamp is unknown or not under the complete control of the experimenter, an empirical expression for the frequency distribution may be used. A convenient expression that approximately represents the line-broadening due to pressure and temperature when the product $k_\nu x$ is small enough to produce measurable absorption only in the central regions of the line is

$$I_\nu = I_0 \exp[-(\omega/\alpha)^2]$$
$$\omega = [2(\nu - \nu_0)/\Delta\nu_D](\ln 2)^{1/2} \quad (17)$$
$$\alpha = \text{emission linewidth/absorption linewidth}$$

then,

$$A_\alpha(k_0x) = \int_{-\infty}^{\infty}\exp[-(\omega/\alpha)^2] \times [1(-\exp(-k_0xe^{-\omega^2})]d\omega \Big/ \int_{-\infty}^{\infty}\exp[-(\omega/\alpha)^2]d\omega \quad (18)$$

The lamp can show self-absorption too. This absorption often occurs in the outer regions of the lamp and may be treated by considering the light to pass through an absorbing layer of neutral atoms. If this layer consists of N_0 atoms/cm^2, then the transmission of the lamp is

$$T_1 = I_0[1 - A(N_0)] \quad (19)$$

and the light that is transmitted through an additional absorbing layer of density N_A atoms/cm^2 is

$$T_2 = I_0[1 - A(N_0 + N_A)] \quad (20)$$

The measured absorption is

$$\Delta = \frac{T_1 - T_2}{T_1} = \frac{A(N_0 + N_A) - A(N_0)}{1 - A(N_0)} \quad (21)$$

For a source that emits several spectral lines,

$$I_0 = \sum_j I_0^j \quad (22)$$

$$k_0^j = \frac{1}{\Delta\nu_D}\left(\frac{\ln 2}{\pi}\right)^{1/2}\frac{\lambda_{0j}^2}{4\pi}\left(\frac{g_2}{g_1}\right)_j\frac{h_jN_T}{\tau_j} \quad (23)$$

with N_T atoms/cm^3 of absorbing medium.

h_j is the fraction of atoms that can absorb a quantum of wavelength λ_{0j}:

$$A_\alpha(N_Tx) = \frac{1 - \sum_j\left\{I_0^j\int\exp\left[-\left(\frac{\omega}{\alpha}\right)^2\right]\exp\left[-K_0\left(\frac{g_2}{g_1}\right)_j h_jN_Txe^{-\omega^2}\right]\right\}d\omega}{\sum I_0^j\int\exp\left[-\left(\frac{\omega}{\alpha}\right)^2\right]d\omega} \quad (24)$$

for the hyperfine structure components of a line

$$k_0{}^j = K_0(g_2/g_1)_j h_j N_T \qquad (25)$$

$$K_0 = \frac{1}{\Delta\nu_D}\left(\frac{\ln 2}{\pi}\right)^{1/2} \frac{\lambda_0{}^2}{4\pi}\frac{1}{\tau_0} \qquad (26)$$

$(g_2/g_1)_j$ and h_j may be calculated for each component. If α is known, the absorption may be obtained from the preceding expression. In practice α often differs from unity and certainly is a function of the operating conditions of the light source. The ratio α and the lamp self-absorption coefficient $N_T x$ may be obtained by empirically fitting an experimentally obtained absorption curve.

V. Experiment

An experimental program was initiated to determine the feasibility of using optical absorption techniques to measure a neutral atom flux in the presence of a large number of excited atoms.

First a lamp detector system was developed and an absorption curve measured by means of a controllable absorption cell. Then the mercury system was mounted on a working ion engine developed by NASA Lewis Research Center.[4, 5] The propellant utilization was determined by measuring the transmission of the Hg 2537-Å line through the exhaust plasma, first with the engine running normally and then with only the engine boiler hot.

The cesium optical system was mounted on a simulated cesium engine. A cesium atomic beam was produced by a temperature controlled atomic oven. From the absorption produced by this atomic beam the total flux from the oven was calculated and compared to the predicted value.

A. Mercury Optical System

The mercury lamp consisted of a small quartz bulb containing a droplet of mercury and a buffer gas of neon. The discharge was electrodeless and excited at 25 Mc. The bulb was protected from air currents and changes in the ambient air temperature by an aluminum enclosure that also provided collimation of the light. The intensity of the light was very stable for periods of many hours and the light showed very little, if any, self-absorption.

The detector was an RCA 7200 photomultiplier tube. A filter, transparent only in the vicinity of the 2537-Å line, was mounted just before the phototube. A collimating system was mounted in front of the filter to prevent stray light from striking the photocathode and to define an optical path. The anode voltage signal was displayed on an oscilloscope using a differential comparator preamp.

In order to distinguish that part of the phototube signal due to the lamp from the signal due to extraneous light produced by the engine, a light chopper was installed in front of the lamp. Light from the lamp then consisted of pulses 1 msec long spaced 12 msec apart.

The lamp detector system was mounted on an optical bench and the transmission of the Hg 2537-Å line was measured as a function of the density of mercury atoms. The absorbing layer of atoms was provided by an absorption cell between the lamp and the detector. Figure 5 shows the experimental configuration.

The absorption cell was evacuated and contained a small amount of distilled mercury. The walls of the cell were heated and the mercury vapor pressure controlled by a relatively cool tungsten rod that passed through the glass wall of the mercury reservoir. The vapor pressure was calculated from the rod temperature using published tables.[6] Several independent absorption measurements were in excellent agreement, indicating the temperature of the rod was known to within a fraction of a degree centigrade. Figure 6 shows the experimental absorption curve and two theoretical absorption curves with $N_0 = 0$, for $\alpha = 1.5$ and $\alpha = 2.0$. In the theoretical calculation, the hyperfine structure was assumed to consist of 5 separate lines of equal intensity, so that $h = \frac{1}{5}$ for each line. The absorption constant K_0 was obtained from the known lifetime[3] of the $^3P_1$ state ($\tau = 1.1 \times 10^{-7}$ sec). This yields $K_0 = 6.56 \times 10^{-13}$ cm^3/atom. The constant α is the ratio of the lamp linewidth to the absorber linewidth and is a function of the temperature of both. Alpha decreases by about 7% over the range plotted. The agreement with experiment is satisfactory for $\alpha = 2$ and would be improved if its temperature dependence were considered.

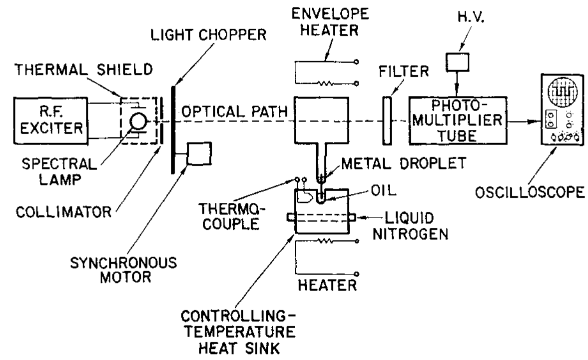

Fig. 5 Diagram of the optical absorption system.

The mercury lamp detector system was mounted on an electron bombardment ion thruster developed by NASA Lewis Research Center.[4,5] As shown in Fig. 7, light from the mercury lamp was reflected from a front surface mirror through a quartz window, into the vacuum chamber, through the exhaust of the engine, through another quartz window, and then reflected by a second mirror into the detector. In order to keep to a minimum the light from the engine, a tight collimation system was established between the entrance and exit windows. In addition, baffles were placed to prevent backstreaming of mercury atoms. The baffle and collimator systems were cooled to liquid nitrogen temperature.

After the engine ran a few seconds, a light-attenuating film was noticed on the windows. A remotely controlled shutter system was therefore installed in front of each window. A comparison of the transmission to the calibration curves

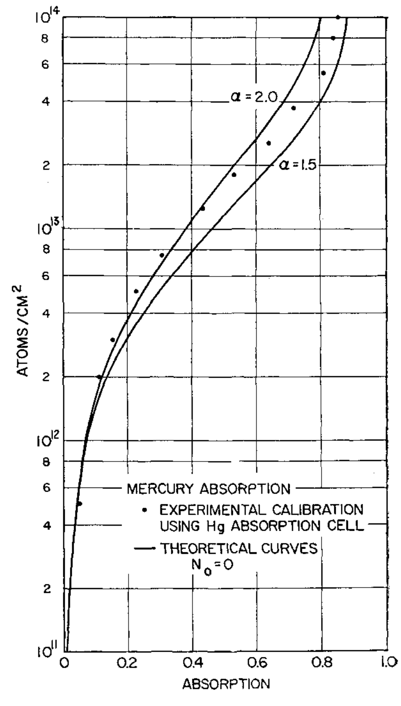

Fig. 6 Absorption of the mercury 2537-Å line.

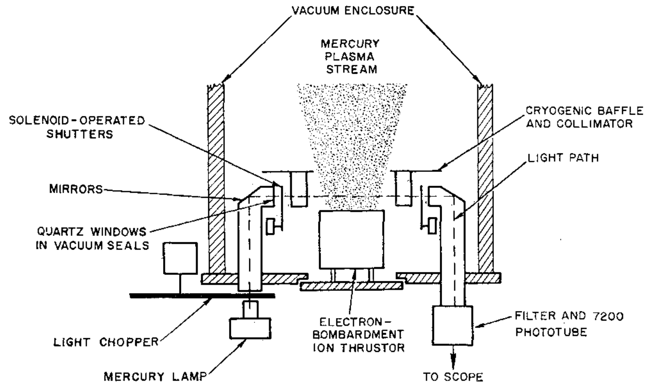

Fig. 7 Optical absorption system used with ion thruster.

yields the number of neutral atoms present in the optical path. The ratio of neutral atoms in the exhaust plasma under running conditions to that emitted by the boiler yields the propellant utilization efficiency.

The propellant utilization may be obtained gravimetrically by running the engine at a constant boiler temperature for a considerable length of time, usually a few hours. The mass lost by the boiler is then the average rate of emission of atoms times the running time. A comparison of this rate with the plasma current corrected for multicharged ions yields the utilization efficiency. Table 3 shows a comparison of the two results. At a boiler temperature of 350°F and with a plasma current of 235 ma, the resonance radiation emanating from the engine saturated the detector.

B. Cesium Optical System

A cesium lamp detector system similar to that for mercury was mounted on an optical bench and an absorption curve for the 8523-Å line was measured.

The cesium lamp was Spectral Model X49-609, manufactured by Varian Associates, and is similar to the alkali-metal spectral lamp developed by Bell, Bloom, and Lynch.[7] It consists of a bulb containing cesium vapor that is excited at 130 Mc and a housing to provide thermal stabilization. Compared to the mercury lamp, the particular lamp used proved to be relatively unstable even for short periods of time. During the absorption cell calibration experiment, the lamp was monitored by adding a small mirror to the lamp side of the chopping wheel and reflecting a pulse of light around the absorption cell into the detector.

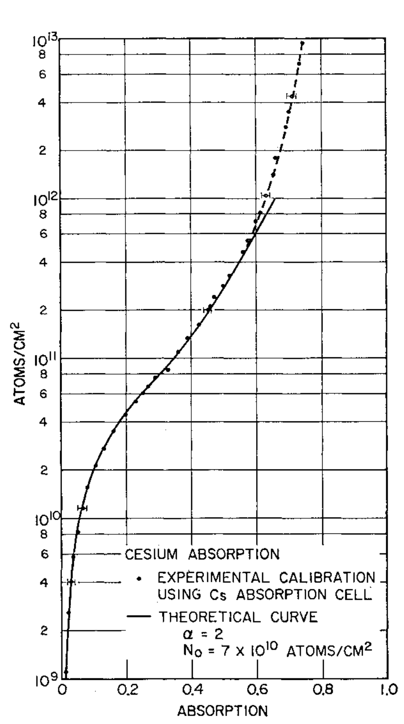

Fig. 8 Absorption of the cesium 8523-Å line.

The detector was an RCA 7102 photomultiplier tube. A filter with an approximately 50-Å wide window at about 8525-Å and a collimating system were mounted in front of the phototube. The light chopper and an absorption cell similar to that of the mercury experiment were also used. The cesium vapor pressure was calculated with the Langmuir[8] formula:

$$\log_{10}P(\text{mm Hg}) = 11.0531 - 4041/T - 1.35 \log_{10}T \quad (27)$$

The cesium absorption curve revealed that the lamp also showed considerable self-absorption. Repeated experiments indicated that the amount of self-absorption varied with ambient room temperature and the length of time the lamp had been on. This necessitated that an absorption cell be part of the lamp detector system and that the system be calibrated each time it was used. Figure 8 shows the experimental absorption curve and the theoretical curve with $\alpha = 2$ and $N_0 = 7 \times 10^{10}$. The theoretical curve assumes that each fine structure line is distinct with negligible overlapping. The relative intensity of each line of the hyperfine group is given in Table 1. The absorption constant K_0 was calculated from the known lifetime[3] of the excited state $\tau = 3.3 \times 10^{-8}$ sec, yielding

$$K_0 = 2.2 \times 10^{-11} \text{ cm}^2/\text{atom}$$

An atomic cesium beam was created by cesium atoms effusing through a grid of small holes on the end of a copper boiler that was maintained at a uniform temperature. The grid consisted of 706 holes of 0.010-in.-diam photo etched in 0.003-in.-thick stainless steel, and was heated to a much higher temperature than the boiler to prevent any trapping of cesium atoms. Baffles cooled by liquid air trapped any cesium not in the beam itself. Figure 9 illustrates the cesium atomic beam system.

A simple calculation shows that the number of atoms per square centimeter of light beam cross section, measured by resonance absorption of the light that passes through this atomic beam, is

$$N_0 = (S_g/2\pi)(n/Z) \sin\phi \quad (28)$$

with

S_g = total area of the holes
Z = distance of light path from the holes
ϕ = maximum angular divergence of the atomic beam (here limited by a collimator)
n = atom density within the boiler

For the experimental geometry used,

$$N_0 = 6.03 \times 10^{-3} n S_g (\text{atoms/cm}^2)$$

The atomic flux is given by[8]

$$N = \tfrac{1}{4} n \bar{v} S_g (\text{atoms/sec}) \quad (29)$$

with $\bar{v}$ the mean molecular velocity, so that

$$N = 41.6 \bar{v} N_0 (\text{atoms/sec}) \quad (30)$$

Figure 10 compares the flux obtained from the optically measured cesium density to the flux calculated from the effu-

Table 3 Results

Boiler temp.	Plasma current	Corrected equivalent singly ionized current	Propellant Utilization: Gravimetrically determined	Propellant Utilization: Optically measured
300°F	115 ma	100 ma	89%	94 ± 5%
325°F	190 ma	165 ma	83%	80 ± 5%

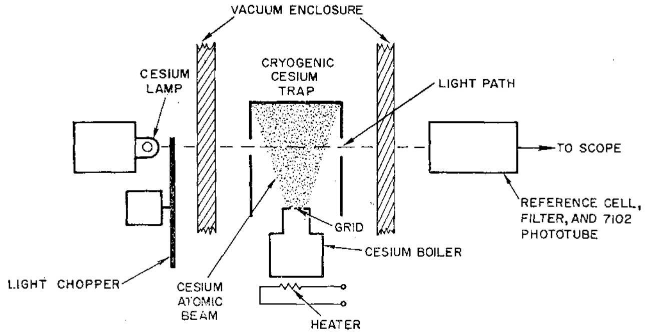

Fig. 9 Optical absorption system used with the cesium atomic beam.

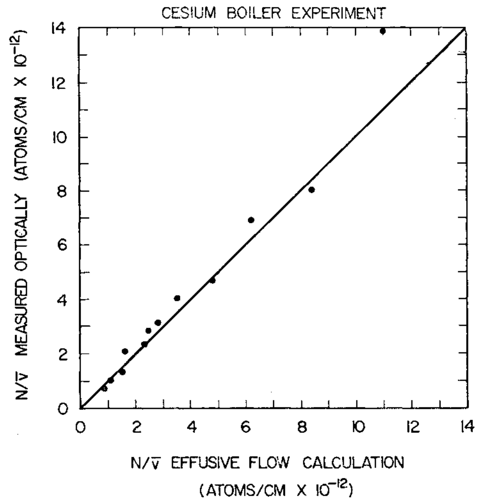

Fig. 10 Results of the cesium atomic beam experiment.

sion model. The effusion model calculation includes a geometrical transmission factor of $\frac{7}{10}$ for the grid. The experimental points are shown with a straight line of 45° slope.

VI. Application to Ion Thrusters

For application purposes, the absorption of resonance radiation may be considered a function of the integral of the neutral atom density N_T over the optical path length L, or, in other words, as the number of atoms per unit area of cross section of the light beam. An estimate of the sensitivity of the technique may be obtained by using the value of 10% absorption as a convenient number. An absorption of 10% can be measured with approximately 10% precision. The values of N_TL which yield a 10% absorption are, respectively, 2×10^{10} atoms/cm^2 for cesium and 1.5×10^{12} atoms/cm^2 for mercury.

A practical engineering relationship is the length of absorbing light path required to achieve, say, 10% absorption from a given ion beam source. Using the relationship that the flux equals the product of particle density times particle velocity, surface flux density may be derived from the measured charge density by assuming an appropriate average streaming velocity. For a cesium beam at 1,500°K, one may assume that the average velocity of the neutral atoms is approximately 3×10^4 cm/sec. Then, in order to obtain 10% absorption, the product of surface flux and path length for cesium must be

$$jL \simeq 0.1 \text{ ma/cm}^2 \text{ equivalent}$$

For example, the optical absorption technique would show 10% absorption when used with a cesium ion source operating at 10 ma/cm^2 of ions and 5% neutrals, if the source were only 2 mm wide. At 1% neutrals, the required source size would be 1 cm.

With a mercury atomic beam at 500°K, the required value for 10% absorption is

$$jL \simeq 3.5 \text{ ma/cm}^2 \text{ equivalent}$$

When used with a mercury thruster operating at 3 m/cm^2 of ions and 90% propellant utilization, an absorption of 10% would be seen if the light path length were 10 cm.

These examples assume single traversal of the atomic beam by the light rays. An additional sensitivity can be attained, of course, through multiple traversal, at the expense of additional optical complication.

VII. Summary and Conclusions

Absorption curves have been measured for both mercury and cesium optical resonance systems. These experimental absorptions may be explained by a model that takes into consideration the hyperfine structure of the resonance lines, line broadening and self-absorption in the source of radiation, and the Doppler broadening of the absorbing atoms.

The flux from a cesium boiler was optically determined from a comparison between the absorption produced by the flux and that produced by an absorption cell.

A resonance lamp detector system was mounted on a mercury engine and the propellant utilization efficiency determined optically. The values obtained by resonance absorption agreed with those obtained by gravimetric means.

In applying the resonance probe to a working thruster, considerable care must be exercised to prevent optical surfaces from being coated and thereby yielding erroneous absorption. In addition, the photodetector must be shielded from the light produced in the engine and its exhaust. The coating of optical surfaces may be reduced by baffles and appropriate shuttering techniques. The effect of the engine's background light can be minimized through narrow collimation of the lamp signal and by employing a lamp of high intensity. All factors that may affect the intensity of the lamp must be carefully controlled, and, if possible, the lamp intensity should be monitored.

In conclusion, both the mercury and the cesium resonance probes were shown to be sufficiently sensitive to measure the neutral effluxes from practical ion propulsion devices, thus adding to the inventory of tools and techniques by which the performance of these thrusters may be diagnosed.

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